QUICK REVIEW 5.2 *(For help, go to Section 5.1.)*

In Exercises 1–6, write the expression in terms of sines and cosines only. Express your answer as a single fraction.

1. $\csc x + \sec x$

2. $\tan x + \cot x$

3. $\cos x \csc x + \sin x \sec x$

4. $\sin \theta \cot \theta - \cos \theta \tan \theta$

5. $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x}$

6. $\dfrac{\sec \alpha}{\cos \alpha} - \dfrac{\sin \alpha}{\csc \alpha \cos^2 \alpha}$

In Exercises 7–12, determine whether or not the equation is an identity. If not, find a single value of x for which the two expressions are not equal.

7. $\sqrt{x^2} = x$

8. $\sqrt[3]{x^3} = x$

9. $\sqrt{1 - \cos^2 x} = \sin x$

10. $\sqrt{\sec^2 x - 1} = \tan x$

11. $\ln \dfrac{1}{x} = -\ln x$

12. $\ln x^2 = 2 \ln x$

SECTION 5.2 EXERCISES

In Exercises 1–4, prove the algebraic identity by starting with the LHS expression and supplying a sequence of equivalent expressions that ends with the RHS expression.

1. $\dfrac{x^3 - x^2}{x} - (x - 1)(x + 1) = 1 - x$

2. $\dfrac{1}{x} - \dfrac{1}{2} = \dfrac{2 - x}{2x}$

3. $\dfrac{x^2 - 4}{x - 2} - \dfrac{x^2 - 9}{x + 3} = 5$

4. $(x - 1)(x + 2) - (x + 1)(x - 2) = 2x$

In Exercises 5–10, tell whether or not $f(x) = \sin x$ is an identity.

5. $f(x) = \dfrac{\sin^2 x + \cos^2 x}{\csc x}$

6. $f(x) = \dfrac{\tan x}{\sec x}$

7. $f(x) = \cos x \cdot \cot x$

8. $f(x) = \cos (x - \pi/2)$

9. $f(x) = (\sin^3 x)(1 + \cot^2 x)$

10. $f(x) = \dfrac{\sin 2x}{2}$

In Exercises 11–51, prove the identity.

11. $(\cos x)(\tan x + \sin x \cot x) = \sin x + \cos^2 x$

12. $(\sin x)(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

13. $(1 - \tan x)^2 = \sec^2 x - 2 \tan x$

14. $(\cos x - \sin x)^2 = 1 - 2 \sin x \cos x$

15. $\dfrac{(1 - \cos u)(1 + \cos u)}{\cos^2 u} = \tan^2 u$

16. $\tan x + \sec x = \dfrac{\cos x}{1 - \sin x}$

17. $\dfrac{\cos^2 x - 1}{\cos x} = -\tan x \sin x$

18. $\dfrac{\sec^2 \theta - 1}{\sin \theta} = \dfrac{\sin \theta}{1 - \sin^2 \theta}$

19. $(1 - \sin \beta)(1 + \csc \beta) = 1 - \sin \beta + \csc \beta - \sin \beta \csc \beta$

20. $\dfrac{1}{1 - \cos x} + \dfrac{1}{1 + \cos x} = 2 \csc^2 x$

21. $(\cos t - \sin t)^2 + (\cos t + \sin t)^2 = 2$

22. $\sin^2 \alpha - \cos^2 \alpha = 1 - 2 \cos^2 \alpha$

23. $\dfrac{1 + \tan^2 x}{\sin^2 x + \cos^2 x} = \sec^2 x$

24. $\dfrac{1}{\tan \beta} + \tan \beta = \sec \beta \csc \beta$

25. $\dfrac{\cos \beta}{1 + \sin \beta} = \dfrac{1 - \sin \beta}{\cos \beta}$

26. $\dfrac{\sec x + 1}{\tan x} = \dfrac{\sin x}{1 - \cos x}$

27. $\dfrac{\tan^2 x}{\sec x + 1} = \dfrac{1 - \cos x}{\cos x}$

28. $\dfrac{\cot v - 1}{\cot v + 1} = \dfrac{1 - \tan v}{1 + \tan v}$

29. $\cot^2 x - \cos^2 x = \cos^2 x \cot^2 x$

30. $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$

31. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

32. $\tan^4 t + \tan^2 t = \sec^4 t - \sec^2 t$

33. $(x \sin \alpha + y \cos \alpha)^2 + (x \cos \alpha - y \sin \alpha)^2 = x^2 + y^2$

34. $\dfrac{1 - \cos \theta}{\sin \theta} = \dfrac{\sin \theta}{1 + \cos \theta}$

35. $\dfrac{\tan x}{\sec x - 1} = \dfrac{\sec x + 1}{\tan x}$

36. $\dfrac{\sin t}{1 + \cos t} + \dfrac{1 + \cos t}{\sin t} = 2 \csc t$

37. $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{2 \sin^2 x - 1}{1 + 2 \sin x \cos x}$

38. $\dfrac{1 + \cos x}{1 - \cos x} = \dfrac{\sec x + 1}{\sec x - 1}$

39. $\dfrac{\sin t}{1 - \cos t} + \dfrac{1 + \cos t}{\sin t} = \dfrac{2(1 + \cos t)}{\sin t}$

40. $\dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

41. $\sin^2 x \cos^3 x = (\sin^2 x - \sin^4 x)(\cos x)$

42. $\sin^5 x \cos^2 x = (\cos^2 x - 2 \cos^4 x + \cos^6 x)(\sin x)$

43. $\cos^5 x = (1 - 2 \sin^2 x + \sin^4 x)(\cos x)$

44. $\sin^3 x \cos^3 x = (\sin^3 x - \sin^5 x)(\cos x)$

45. $\dfrac{\tan x}{1 - \cot x} + \dfrac{\cot x}{1 - \tan x} = 1 + \sec x \csc x$

46. $\dfrac{\cos x}{1 + \sin x} + \dfrac{\cos x}{1 - \sin x} = 2 \sec x$

47. $\dfrac{2 \tan x}{1 - \tan^2 x} + \dfrac{1}{2 \cos^2 x - 1} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

48. $\dfrac{1 - 3 \cos x - 4 \cos^2 x}{\sin^2 x} = \dfrac{1 - 4 \cos x}{1 - \cos x}$

49. $\cos^3 x = (1 - \sin^2 x)(\cos x)$

50. $\sec^4 x = (1 + \tan^2 x)(\sec^2 x)$

51. $\sin^5 x = (1 - 2 \cos^2 x + \cos^4 x)(\sin x)$

In Exercises 52–57, match the function with an equivalent expression from the following list. Then confirm the match with a proof. (The matching is not one-to-one.)

(a) $\sec^2 x \csc^2 x$ **(b)** $\sec x + \tan x$ **(c)** $2 \sec^2 x$

(d) $\tan x \sin x$ **(e)** $\sin x \cos x$

52. $\dfrac{1 + \sin x}{\cos x}$

53. $(1 + \sec x)(1 - \cos x)$

54. $\sec^2 x + \csc^2 x$

55. $\dfrac{1}{1 + \sin x} + \dfrac{1}{1 - \sin x}$

56. $\dfrac{1}{\tan x + \cot x}$

57. $\dfrac{1}{\sec x - \tan x}$

Standardized Test Questions

58. True or False The equation $\sqrt{x^2} = x$ is an identity. Justify your answer.

59. True or False The equation $(\sqrt{x})^2 = x$ is an identity. Justify your answer.

You should answer these questions without using a calculator.

60. Multiple Choice If $f(x) = g(x)$ is an identity with domain of validity D, which of the following must be true?

I. For any x in D, $f(x)$ is defined.

II. For any x in D, $g(x)$ is defined.

III. For any x in D, $f(x) = g(x)$.

(A) None

(B) I and II only

(C) I and III only

(D) III only

(E) I, II, and III

61. Multiple Choice Which of these is an efficient first step in proving the identity $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$?

(A) $\dfrac{\sin x}{1 - \cos x} = \dfrac{\cos\left(\dfrac{\pi}{2} - x\right)}{1 - \cos x}$

(B) $\dfrac{\sin x}{1 - \cos x} = \dfrac{\sin x}{\sin^2 x + \cos^2 x - \cos x}$

(C) $\dfrac{\sin x}{1 - \cos x} = \dfrac{\sin x}{1 - \cos x} \cdot \dfrac{\csc x}{\csc x}$

(D) $\dfrac{\sin x}{1 - \cos x} = \dfrac{\sin x}{1 - \cos x} \cdot \dfrac{1 - \cos x}{1 - \cos x}$

(E) $\dfrac{\sin x}{1 - \cos x} = \dfrac{\sin x}{1 - \cos x} \cdot \dfrac{1 + \cos x}{1 + \cos x}$

62. Multiple Choice Which of the following could be an intermediate expression in a proof of the identity $\tan \theta + \sec \theta = \dfrac{\cos \theta}{1 - \sin \theta}$?

(A) $\sin \theta + \cos \theta$

(B) $\tan \theta + \csc \theta$

(C) $\dfrac{\sin \theta + 1}{\cos \theta}$

(D) $\dfrac{\cos \theta}{1 + \sin \theta}$

(E) $\cos \theta - \cot \theta$

63. Multiple Choice If $f(x) = g(x)$ is an identity and $\dfrac{f(x)}{g(x)} = k$, which of the following must be *false*?

(A) $g(x) \neq 0$

(B) $f(x) = 0$

(C) $k = 1$

(D) $f(x) - g(x) = 0$

(E) $f(x)g(x) > 0$